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Measurement of Self-Inductance.

By

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DISSERTATION

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The absolute measurement and standards of self inductance has within the last few years become a problem of increasing importance. The best known method and that most generally used is the Rayleigh method. However, this necessitates very intelligent and careful work to obtain results concordant to one percent. What seem to be the most accurate methods are various modifications of the Wheatstone bridge using alternating currents with an electrodynamometer, telephone, or optical telephone as the measuring instrument.

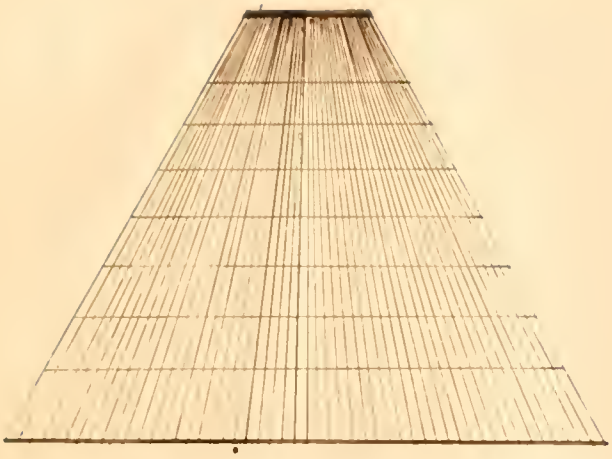
In January 1898 Professor Rowland published a brief description of some twenty-six methods for the measurements and comparison of self-inductance capacity and mutual inductance. These methods are mostly of the Wheatstone bridge type and require the use of an alternating current and an electrodynamometer. As only a few of them have hitherto been tried it was deemed desirable to systematically test them and determine their value, particularly in regard to the measurement of self inductance. Such is the aim of the present investigation.

Source of Current and Frequency determination.

Since the value of the self inductance or capacity as found by any of the methods depends upon the square of the current frequency it was necessary to secure a current of harmonic wave form whose period was as constant as possible, and to devise a method of accurately determining that period at the moment of adjustment. This was very satisfactorily accomplished. In the deduction of the formulae for all the methods the assumption is made that we have a simple harmonic electromotive force. In general an alternating current supplied by a dynamo has not only its fundamental period but also the odd upper partials. However, a generator constructed without too much iron and properly wound, if the field is not very strongly excited will give a very good sine wave; especially if the resistance in the circuit contains self-inductance and no iron. These requirements were well satisfied by the dynamo used, which gave a very good sine wave. The alternator was constructed at the University. Its armature consisted of twelve coils fastened flat on a thin German silver plate revolving between twelve field pieces producing six poles. This dynamo was coupled directly to a direct-

current Crocker Wheeler motor. As long as the load on the dynamo was not varied the speed remained very constant. Greater constancy of speed was sometimes secured in working in the evening when the motor current was supplied from storage batteries. The voltage furnished by the alternator could be controlled by slightly altering the strength of the field. For the determination of the frequency a chronograph was used. A kind of speed counter was connected to the shaft of the dynamo and a contact so arranged that every ninety-seven revolutions of the dynamo armature a circuit was closed and a record made on the chronograph sheet. The chronograph was set up close to the rest of the apparatus and by means of a thread the pen could be raised for an instant from the sheet and so the exact kind of moving an adjustment could be readily noted. By interchanging the gear wheels the cylinder of the chronograph was made to run at a rate three and one-half times the customary speed. The distance between the checks indicating the seconds was so increased to over five and one-half centimeters.

A cable similar to that shown in the figure was then constructed, consisting of fine lines scratched on transparent colluloid and blackened with India ink. By means of



to read the chronograph sheet directly to fiftieths of a second and to estimate to five hundredths. However, in reading a chronograph sheet as exactly as this, certain precautions must be taken. First, the checks indicating the seconds should be sharp and distinct. Secondly, an error may arise if the chronograph does not run quite regularly. This can generally be remedied by properly adjusting the governor. By this method the frequency could be quite accurately determined, to 1/10% or closer, if the sheet was read with sufficient care. It is to be noted that if sidereal seconds are recorded on the chronograph sheet they should be reduced to solar seconds, otherwise an error of 3/10% is introduced in the absolute value of the frequency and therefore of 6/10% in the value of self-inductance.

APPARATUS.

Electrodynamometer. A Rowland electrodynamometer was used. The self-inductance of the fixed coils was .0165 henry, and of the hanging coil .0007 henry.

Resistances. In the measurements of self-inductance with alternating currents, ordinary resistance boxes are of little or no value owing to the electrostatic capacity of the doubly wound coils. Except in the case of a few determinations with Method 23, all the resistances used were made of a special German silver resistance wire, B. & S.gauge, Nos. 30 & 27. This wire was not doubled, but wound continuously in one layer on slates or pieces of micarite, each slate containing about 2000 ohms conveniently subdivided. The self-inductance and capacity of such resistance is entirely negligible. For fine adjustment a small resistance box was constructed differing from an ordinary box in that its coils were wound flat on thin pieces of micarite.

Coils and Condenser. The coils whose inductances were measured were as follows.

P , external diameter 33.[illegible] c. ., internal diameter 23.9 cm.,

5.7237

S. External diameter 23.5 cms., internal diameter 15 cms., depth about 3½ cms. It was made of 2082 turns of No 22 B & S. single cotton covered copperwire. Resistance about 69 ohms. Self-inductance found to be 1.0331 henrys

X, Y & Z. Three specially constructed coils described later.

Condenser: A $\frac{1}{3}$ microfarad mica condenser.

Conditions of sensibility.

The deflection of an electrodynamometer is proportional to the product of the currents flowing in the fixed and movable coils multiplied by the cosine of the phase difference of the two currents. In most of these Borda methods the adjustment consists in an alteration of inductive resistances in one or more branches of the bridge that there is a maximum three phase difference between the currents in the fixed and movable coils respectively. Evidently in order to secure maximum sensibility the currents through the dynamometer coils should be as large as possible, heating alone being the limit.

Sources of Error. In work with alternating currents great care must be exercised that one part of the network does not exert an inductive action on another e. g. that the coil whose inductance is being measured does not affect the movable coil of the electrodynamometer. Induction was carefully guarded against by the arrangement and tested for by means of reversing switches. The electrostatic capacity of coarsely wound coils and resistances has already been considered as well as the precautions taken to avoid it. In the dynamometers employed all the connections were made as short as possible and the wires very twisted. Heating of the resistance was avoided as much as possible and in the very accurate work

the resistances of the several arms of the network were measured after adjustments on a Nat'l Bur. standard "Post Office" box. All resistances are in International ohms.

METHODS USED.

I.

Method 25. The first method tried was that designated in Professor Rowland's article as number 25. It is the alternate method intended for the measurement of either self induction or capacity, the value of which depends upon the square of the current frequency. It was given a thorough test by Mr. T. D. Penniman*, except in so far as his method of measuring the frequency was rough compared with that used in this investigation, and he ascribes the lack of uniformity of his results chiefly to a want of knowledge of the current period. It was therefore thought well to give the method a second trial in regard to the measurement of self induction. In this method the bridge coil is shunted off the fixed coils circuit and the deflection with a non-inductive resistance in circuit

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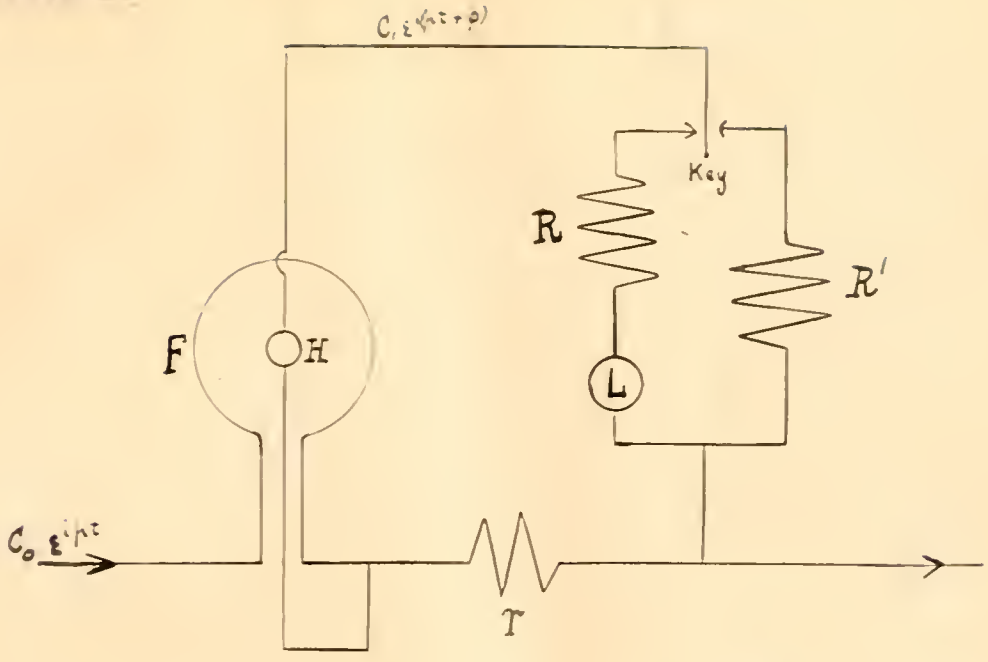
* Am. Journ. Sci. VIII. p. 37, 1899.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R'
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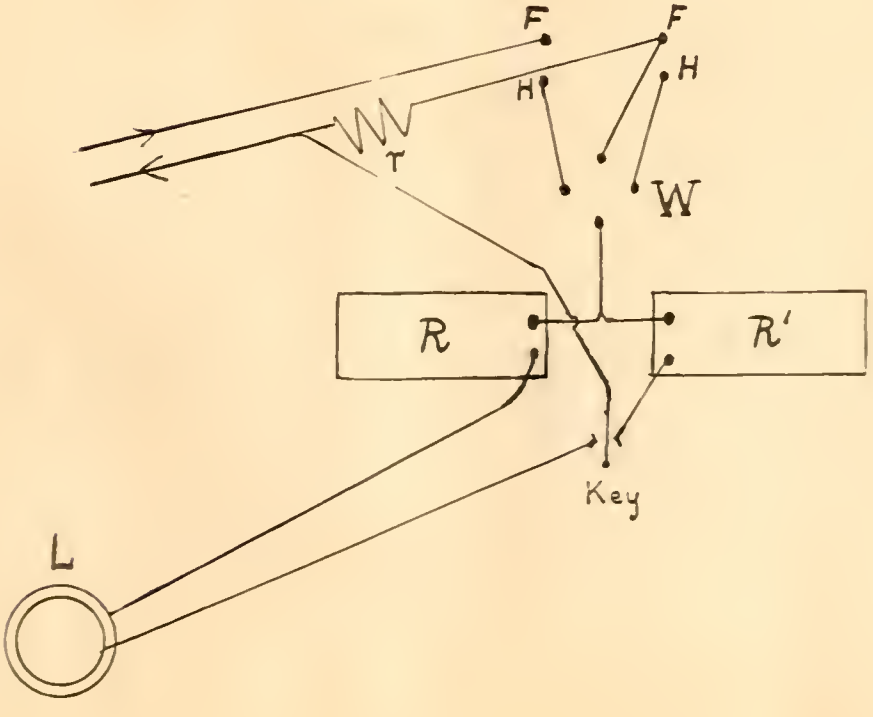
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with the unknown coil in the lead as that of an inductive resistance in circuit with the unknown coil. The method and connections are shown in the figure, F and H represent the fixed and moving coils of the electrodynamometer; R, R' and r, the total resistances of the three branches; L is the self-inductance of the coil to be measured, W is a reversing commutator. The formula for the method, as reduced by Mr. Perkin, is,

$$p^2 L^2 = (R' - R) (R + r)$$

Where p is equal to 2π multiplied by the current frequency.

This method did not prove satisfactory for measuring self-inductance. With care it is possible to obtain values which do not differ among themselves by more than 1%. One drawback to the method is its lack of sensibility, that is it is not usually possible to adjust the resistances closer than one part in three hundred. Again, since the electrodynamometer was very "dead beat" it required a certain small time to compare two deflections, to be sure, not long, but enough for the current period to alter slightly unless special precautions were taken. A few determinations of the self-inductance of coil C and C were given. It will be noticed that the values are lower than those obtained later by more satisfactory methods.

This is probably due to the use of ordinary resistance coils possessing electrostatic capacity. This method was the first tried and is the only one in which such resistances were used.

Table I. Coil C.

Zero	Deflection	γ	R	R'	n	L
249.8	153.0	4.017	149.7	1513.7	57.567	1.266 henry
"	154.9	"	"	1531.7	57.227	1.282 "
248.2	151.8	"	"	1518.7	57.453	1.271 "
"	151..	"	"	1530.7	57.510	1.275 "
247.7	127.0	"	199.7	1241.7	57.453	1.276 "
247.3	126.2	"	"	1238.7	57.510	1.273 "

Mean 1.274 "

Table II. Coil C.

Zero	Deflection	r	R	R'	n (cycles)	L
240.0	158.2	4.017	141.5	1300.7	63.885	1.023 henry
"	158.5		"	1283.7	63.62	1.021 "
"	"		"	1283.7	63.329	1.020 "
239.2	301.0		"	1288.7	63.261	1.028 "
"	"		"	1281.7	"	1.025 "
239.0	134.2		191.5	1041.7	64.095	1.012 "
"	134.8		"	1038.7	63.955	1.013 "
238.7	345.8		"	1036.7	"	1.012 "
"	"		"	1038.7	"	1.013 "
249.0	121.3		241.5	920.7	64.095	1.014 "
"	"		"	922.7	"	1.015
239.1	129.9		291.5	850.7	64.167	1.009
"			"	"	64.095	1.010
239.7			491.5	836.7	64.23	1.025

Mean 1.0171 "

II.

Method 11. The first method tested was that given in Professor Powell's paper No. number 11. It is an absolute method for measuring either self inductance or capacity. It is a zero deflection method depending upon adjusting the resistances until there is a ninety degree difference in phase between the currents flowing in the fixed and hanging coils of the electrodynamometer. The method was devised and first used by Oberbeck* , later by Troje**. However, the frequency determination of both these investigators was quite crude and , as will be seen from the formula, the value of L or C varies as the square of the current period. Diagrams of the method and connections are given in the figure. As before F and H represent the fixed and hanging coils of the electrodynamometer ; R_1 , R_{11} , R' , R" and r are the entire resistances of the several branches, L is the self inductance of the coil to be measured and l , that of the hanging coil of the electrodynamometer, W is a commutator for reversing the current through the hanging coil, S is a switch by means of which the current could be sent through a resistance X, in value closely equal to

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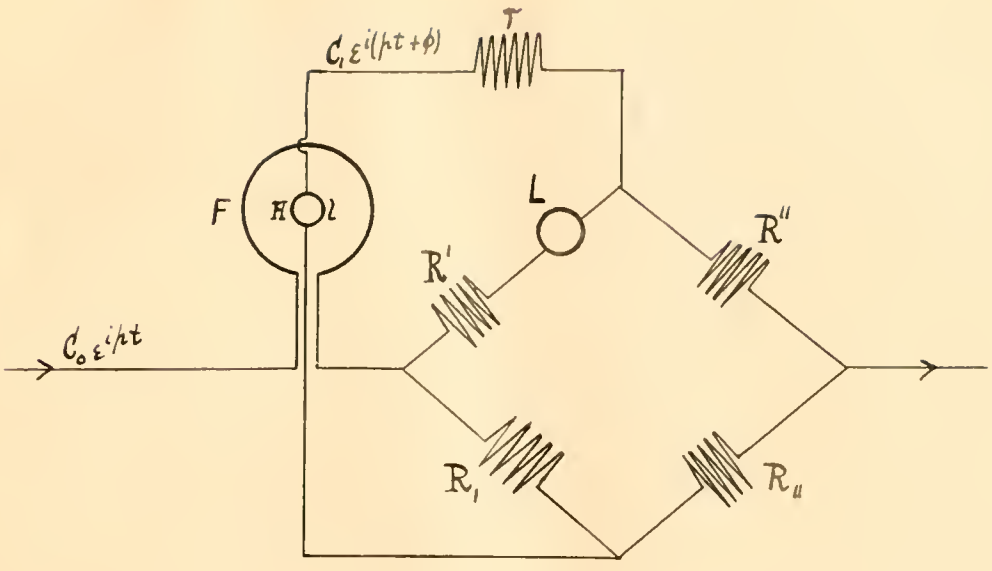
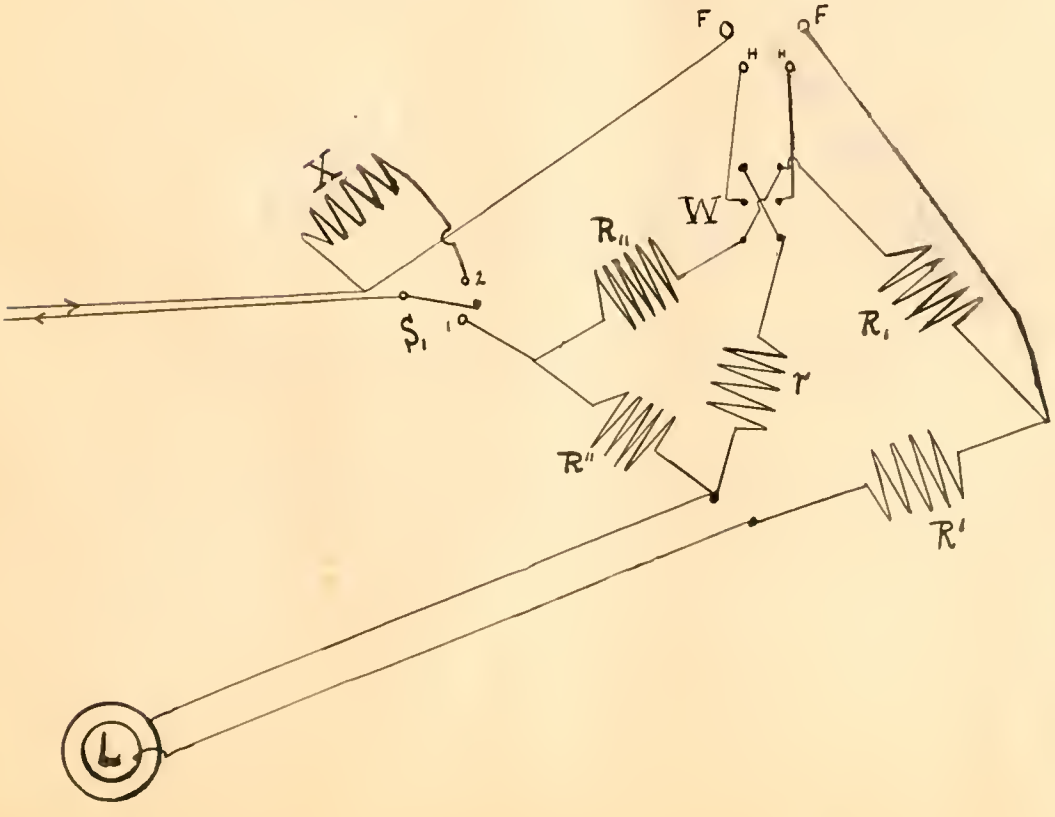
* Wied. Ann. #17, 1882, p. 816.
** " " #19, 1883, p. 201.

Method 14


T
$C_1 \varepsilon^{i(pt+\phi)}$
F
H
l
L
R'
R''
$C_0 \varepsilon^{ipt}$
R_I
R_{II}


F
F
H
H
X
W
R_{II}
2
S_1
1
R_I
r
R''
R'
L

the impedance of the battery. The coil L to avoid any possible effect on the rest of the network or moving coil of the electrodynamometer was placed at a distance away usually at a level with the moving and fixed coils and perpendicular to them.

If now an alternating electromotive force is applied to the terminals A & B, we may express the currents in the fixed and moving coils of the electrodynamometer by $C_0 \varepsilon^{ipt}$ and $C_1 \varepsilon^{i(pt+\phi)}$, p being equal to 2π times the frequency. By the application of Kirchhoff's laws to the Wheatstone bridge using "generalized resistances", we obtain as the ratio of the current in the battery arm to the current in the bridge arm,

$$\frac{C_0}{C_1}\varepsilon^{i\phi} = \frac{(r+ipl)(R_1 + R_{11} + R' + R'' + ipL)(R_{11} + R'')(R_1 + R' + ipL)}{R_1 R'' - R_{11} R' - i\,p\,L\,R''}$$

Multiplying both sides of this expression by C_1^2, rationalizing, and equating the real parts we obtain,

$$C_0 C_1 \cos\phi = C_1^2 \frac{[R_1 R'' - R_{11} R'][r(R_1 + R_{11} + R' + R'') - p^2 LL]}{[R_1 R'' - R_{11} R']^2 + p^2 L^2 R''^2} + \frac{[R' + R_1][R_{11} + R'']}{}$$

$$- C_1^2 \frac{p^2 L\, l\, R_{11}(R_1 + R_{11} + R' + R'') - p^2 L^2 r R_1 - p^2 L^2 (R + R'') R_1}{[R_1 R'' - R_{11} R']^2 + p^2 L^2 R''^2}$$

This expression must be equal to zero since we adjust the bridge until $\phi = 90^\circ$, therefore $\cos\phi = 0$. Equating to zero and simplifying,

$$L^2 + L l \left[\frac{(r'' + R_{II})(R_I + R_{II})}{R_{II}(r + R'' + R_I)} \right]$$

$$\frac{R'' R_I - R_{II} R'}{p^2 R_I} \left[\frac{r(R_I + R_{II} + R' + R'') + (R' + R_I)(R'' + R_{II})}{r + R'' + R_{II}} \right]$$

This formula is somewhat simplified if R_I is chosen equal to R''. However, for accurate work the equality must be exact. Since the self inductance of the hanging coil of the Rowland electrodynamometer is so small, only .0007 henry, the correction of L due to it is very small, in the case of coil C amounting to .0004 henry.

In using the method the numerical work involved in the calculation of L is rendered somewhat easier if we write the formula,

$$L + sLl = t$$

when very approximately since l is so small,

$$L = t - \frac{s}{t} l$$

where

$$S = \frac{(R'' + R_{II})(R_I + R_{II})}{r + R_I + R''}$$

and $t = \frac{1}{p^2}\left[R_I \frac{R''}{R_I} - R' \right]\left[R' + R_I + \frac{r(R_{II} + R'')}{r + R_{II} + R''} \right]$

This method has found it to be excellent. It is very sensitive and very accurate provided certain precautions

table. [illegible] difficulty was experienced at first from the fact that the values of the self inductance of a coil measured on any one day, although agreeing pretty well among themselves, differed from those obtained at a different time. The cause of this variance proved to be the heating and consequent lack of knowledge of the resistances. In order to secure sensibility of the electrodynamometer it was the custom to make the current through the fixed and moving coils quite heavy. Sometimes the current through the fixed coils was nearly $\frac{1}{10}$ ampere. This was unnecessarily large, as with a current of $\frac{1}{20}$ ampere it is easily possible to adjust the resistances to one part in fifteen hundred or two thousand. If we wish to take exact measurements of inductance the method requires an accurate knowledge of the resistances. To take a specific example, if in the first determination in table IV we increase R or R" by $\frac{1}{20}\%$, a change of -1/5% is introduced in the value of L; similarly changing R or R' by 1/20% we alter L by +1/5%. In general then, L varies only as the first power. Fortunately these errors in L introduced by variation of the resistances are not all of the same sign, and since the current through R must of necessity be quite small

(about $\frac{1}{100}$ ampere) the currents through R + R' and R' + R'' are nearly equal. Hence if we assume an increase of all the resistances of 1/20%, the value of L is increased by about 1 part in 1500. . Therefore, to obtain the best results with the method we must measure the resistances carefully and avoid much heating. To avoid heating of the resistances the following plan of taking readings was adopted. The bridge was first adjusted, this being done by altering R' and R until on reversing the current through the hanging coil of the electrodynamometer there was zero deflection indicating a ninety degree phase difference between the fixed and suspended coils. By means of the switch S (vide fig. p.14) the current was turned through the resistance X, made closely equal to the impedance of the network. This was done to avoid change in the frequency. After a considerable interval, sufficient to avoid the effects of any slight heating in the preliminary adjustment, the current was again switched through the network and R, remaining the same, R' readjusted to give zero deflection, the exact time of adjustment being noted in the chronograph sheet. This adjustment when made would usually hold good for several minutes. However, it was not usually necessary to run the current through the network longer than twenty or thirty seconds. This process was repeated a number of times gi

lonior or larger intervals and took the resistances R' and R, measured. The other arms of the bridge were measured once or twice during the evening. Generally for several successive observations the same value of R' would give zero deflection, R remaining the same. The mean of the frequencies measured for the several observations was taken as being nearest correct.

If, however, R' had to be altered the observations were regarded as separate and so worked out.

It was not the purpose of the author to measure inductance to any great degree of accuracy, but merely to indicate the possibilities of the methods. Resistances were measured only to about two or three parts in ten thousand on the Post Office box which had a considerable temperature correction. Still it will be seen that the determinations of L except those of coil P differ from the mean by less than $\frac{1}{1}$%.

The measurements of coil P were among the first made with this method and the resistances were not measured with as much care as in the case of the other coils

Table III, Coil P

Oct 28th. 1908

$R_{\parallel}$	R''	r	R_1	R'	n	L
103.75	103.67	425.06	900.76	870.92	65.644	.5722 Henrys
..	..	..	..	..	65.614	.5725
..	..	..	948.23	919.72	65.763	.5715
..	..	..	989.3	961.7	65.763	.5720
..	..	..	900.76	870.94	65.614	.5723
..	..	..	948.23	919.67	65.614	.5733
..	..	..	699.9	662.54	65.689	.5698
..	..	..	989.3	961.66	65.614	.5749

Mean .57252

Table IV. Coil C

Dec 10th 1903

Temp	R_{II}	R''	r	R_1	R'	n	L
16.3C	99.907	99.78	968.8	1199.86	1088.6	63.288	1.3044
	after the elapse of some minutes						
	..	..	..	1199.86	1088.6	63.288	1.3044

Dec 13th, 1903

Temp							
15°C	99.94	99.42	968.9	1200.47	1088.0	62.366	1.3055
	..	..	..	..	..	62.446	1.3008
	..	..	..	..	..	62.259	1.3047
		..	..	..	..	62.206	1.3058
						Mean	1.30257

Table V, Coil S.

Nov 13th. 1903

$R_{\parallel}$	R''	r	R_1	R'	n	L
						Henrys
100.0	100.0	425.06	1815.5	1767.2	65.893	1.0309
"	"	"	"	1766.67	65.393	1.0366
"	"	"	1862.9	18155	65.688	1.0306
"	"	"	1904.	1856.96	66.091	1.0311
"	"	"	1904.	1856.96	65.763	1.0353

Mean 1.0332

This does not represent as careful work as Table VI, resistances and frequency were not as accurately known.

Table VI. Col 5

May 13th 1904

$R_{\parallel}$	R''	r	R_1	R'	n	L
						henrys.
100.16	99.42	1020.2	1816.93	1734.0	78.1208	1.0345
..	..	.	..	1734.5	78303	1.0359
	After the elapse of ten minutes					
..	..	..		1734.5	78.278	1.0325
..	..	..	1905.3	1819.3	81.410	1.0330
..	..	..	..	1818.27	81.700	1.0365
..	..	..	..	1848.83	62.604	1.0354
..	..	..	1616.0	152957	76.456	1.0333
..	..	..	..	1528.26	76.068	1.0294
..	..	.	..	1630.1	76.428	1.0299
.	..	.	181693	176064	61.797	1.0307
					Mean	1.0331

The following example shows the numerical work involved by this method

Friday May 13th. - Coil 9.

(a) $R'' = 99.42$ ohms $\quad R_{11} = 100.16$ ohms $\quad r = 1020.2$ ohms

$R' = 1734.0$ " $\quad R_1 = 1816.93$ "

$$W \equiv \frac{r(R_{11}+R'')}{r+R_{11}+R''} = \frac{1020.2 \times 199.58}{1219.78} = 166.92478$$

$$S \equiv \frac{(R_{11}+R'')(R_1+R_{11})}{R_{11}(r+R_{11}+R''} = \frac{199.58 \times 1917.09}{100.16 \times 1219.78} = \frac{382612.822}{122173.1648}$$

$$= 3.1317$$

$l = .0007$ henry , $\frac{S}{2} l = .001096$ henry.

$$t \equiv \frac{1}{p^2}\left[R_1 \frac{R''}{R_{11}} - R'\right]\left[W + R' + R_1\right]$$

$$= \frac{1}{p^2}\left[1816.93 \times \frac{99.42}{100.16} - 1734.0\right][3717.855]$$

$$= \frac{69.506 \times 3717.855}{p^2} = \frac{258413.2296}{p^2} = \frac{508.3436}{p}$$

$n = 78.1208$ cycles , $p \equiv 2\pi n = 490.8486$

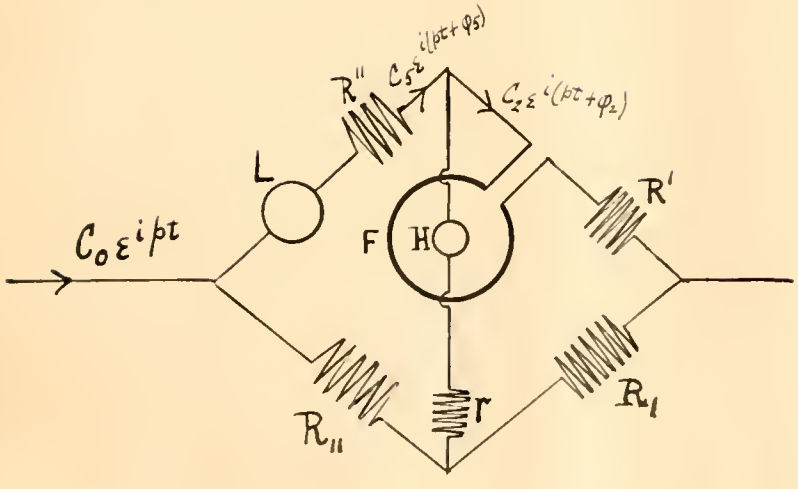
$$t = \frac{508.3436}{490.8486} = 1.03564$$

$L = t - \frac{S}{2} l$ very approximately since l is so small.

$L = 1.03564 - .001096 = 1.0345$ henry

III.

Method 13. Method 13 was next tried. This is also an absolute method for measuring either self-inductance or capacity. It is a zero deflection method depending upon a ninety degree phase difference just as in method 14. In fact, it differs from that method only in that the fixed coils of the electrodynamometer are no longer in the battery arm of the bridge. The connections are shown in the diagram. Neglecting the self-inductance of the fixed and moving coils of the electrodynamometer the formula for the method may be deduced as follows. Proceeding as before we find the ratio of the current in the fixed and moving coils



$$\frac{C_2}{C_3}\varepsilon^{i(\varphi_2-\varphi_3)} = \frac{r(R_{II}+R_1)+R_1(R_{II}+R''+i\rho L)}{R'R_{II}-R_1R''-i\rho R_1 L}$$

Rationalizing the fraction, multiplying through by C_3, and taking the real parts, we have for zero deflection;

$$p^2 L^2 = \frac{[R' R'' - R_1 R''][r(R'' + R_1) + R_1(R'' + R'')]}{R_1^2}$$

We should expect this method not to be as sensitive as Method 14, for the reason that we must in general have a smaller current in the fixed coils of the dynamometer. Moreover since the formula is similar in character to that of Method 14, we would look for the value of L to be affected in the same way by a slight heating of one or more of the resistances. [The formula for Method 14, neglecting ganging coil correction is

$$p^2 L^2 = \frac{R_1 R'' - R' R_1}{R_1(r + R'' + R_1)}\{L + r(R' + R_1 - R_1 + R'') + (R' + R_1)(R'' + R_1)\}$$

The method was set up and some determinations made of the inductance of coil C. The method proved much less sensitive than Method 14, under comparable conditions only about one-half. By sensitive it is meant the degree of accuracy to which it is possible to adjust the resistances.

It is to be noted that the formula as given neglects the self-inductance of the fixed and ganging coils of the electrodynamometer. The exact formula is not difficult to work out but the method is hardly sensitive enough to justify the trouble.

A few measurements of the inductance of coil C were made with more care. The values obtained were too concordant with

[illegible] obviously obtained as well as can be seen from the following example.

R_{II} = 1201.7 R'' = 1503.6 r = 303.91

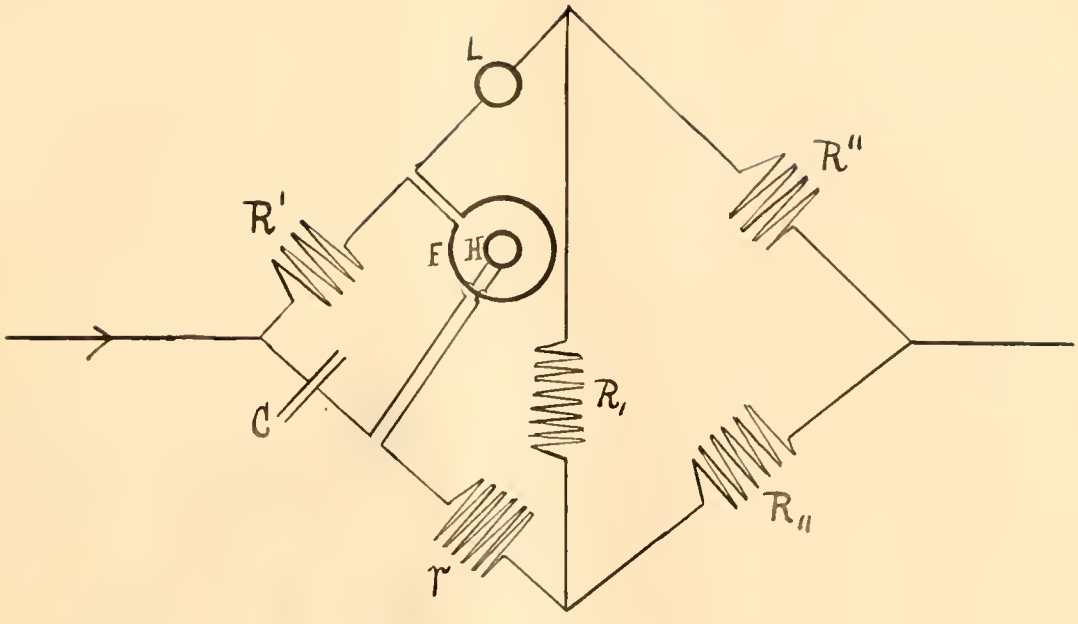
R_{I} = 99.83 R' = 133.33 n = 69.153

L = 1.1483

IV.

Methods 1, 2, 3, etc. Attention was now turned to the first six methods given in Professor Rowland's paper which may be used for the comparison of self inductance with capacity. They are all zero deflection methods depending on a ninety degree phase difference. As will be seen from the formulae the value of $\frac{L}{C}$ is independent of the frequency. Two of these methods have already been used. Method 6 was studied by Mr. Pender but did not yield favorable results. Method 3 was employed by Mr. Potts in the course of some work on electric absorption, and was found very satisfactory. The diagrams and formulae for the methods are as follows. The method of deducing the formulae is the same as that followed in the preceding cases.

Method 2.



$$\frac{L}{C} = \frac{[r(R_{,}+R'') + R_{,,}(r+\quad)][R'(R_{,}+R_{,,}) + R''(R_{,}+R'\quad)]}{(R_{,}+R''+R_{,,})^{2}}$$

The trouble with this method was its lack of sensibility. Various values of resistances and coils were tried, but with the arrangement which was as sensitive as possible the resistances could not be adjusted closer than 1/5%. This, also, when the maximum current to avoid heating was flowing through the network. It will be noted that the formula is rather similar to that of Method 14, so we should expect the value of $\frac{L}{C}$ to be affected in the same way as was L by a shunt circuit in one or more of the resistances. The values of the resistances which seemed to give the most sensitive arrangement

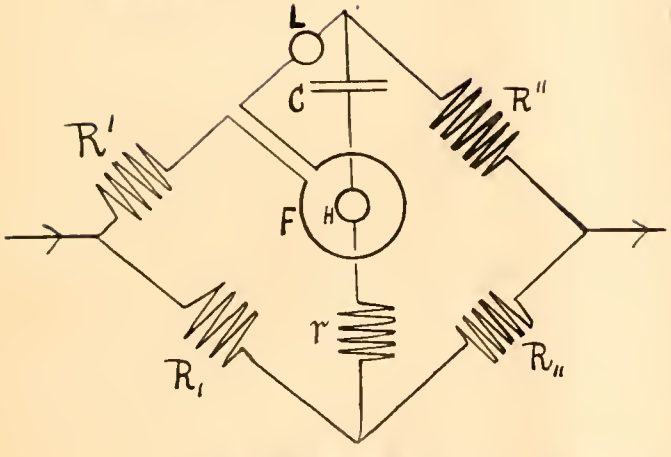
When coil L was compared with a $\frac{1}{3}$ micro-farad mica condenser

were,

R' = [illegible] ohms, R" = 100 ohms = R_{II}

R_I = 1200 ohms r = 5900 ohms.

Method 2.



When L = .570 & C = $\frac{1}{3}$ micro-farad

the values of the resistances which

seemed to give the most sensi-

tive arrangement were,

R" = R_{II} = 100 ohms

R_I = 1200 ohms

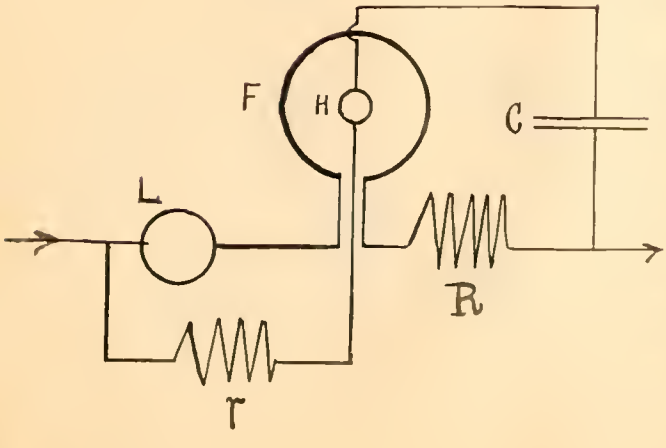
R' = about 1200 ohms.

This method did not seem to be capable of [illegible] much

closer than 1% even when the current through the network was

.06 amp., which was much too large to avoid heating.

Method 3.



This is the simplest of the

methods for determining $\frac{L}{C}$. The

formula is

$$\frac{L}{C} = r_1 r_2.$$

As was already [illegible] stated this

method also has proved very success-

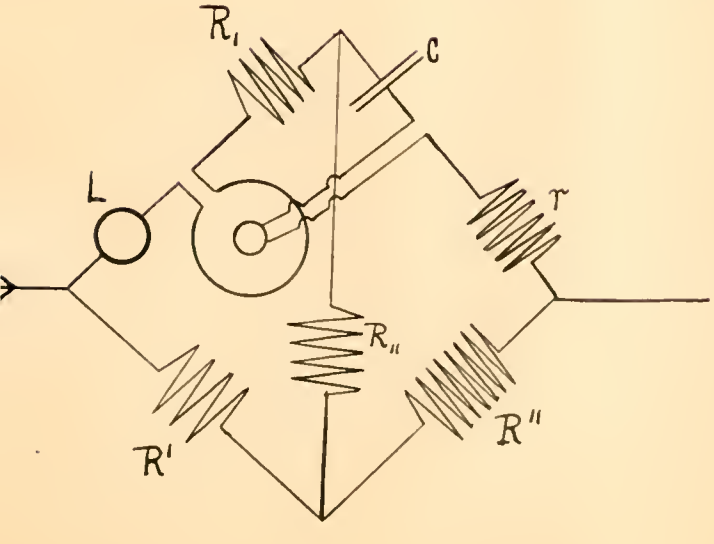
ful. [illegible]

the method a discharge coil P with the $\frac{1}{5}$ microfarad condenser. The method is very sensitive. With r = 1070 ohms and R_{II} = 185 ohms, a change of 2 ohms in r caused a change of 1 mm in the deflection of the electrodynamometer. Determinations of the ratio L / C were not made for the reason that if we wish accurate results it is necessary to measure the absorption resistance of the condenser. This should be done at each determination of L/C as the absorption has been found to vary greatly both with the temperature and the frequency.

The diagram and formulae for methods 4 and 5 are given; also the values of the resistances of the arms which seemed to give the most sensitive arrangement. As before, coil P , was compared with the $\frac{1}{5}$ microfarad condenser.

Method 4.

$$\frac{L}{C} = \left\{R'\left(rr_{II}\right) + R''\left(R'+R_{II}\right)\right\}\left\{\frac{1}{R'R''}\right\}\left\{R'\left(R''+R_{II}\right) + R''\left(R_{II}+R\right)\right\}$$



As adjusted,

R_{II} = 80.24 ohms

R' = 1021.6 ohms

R" = 99.5 "

r = 132.0 "

R = 337.5 "

bits δ_0 = .081 rad. a change of 1 ohms in r is barely detectable and a change of 17 ohms in r' produces only a deflection as measured on the scale of the electrodynamometer.

V.

Methods 7, 8, 9, 10, 11 & 12.

Of the remaining zero reflection methods which depend on a 90 degree phase difference, little needs to be said. Methods 7 and 8 are for determining $\frac{L}{c}$ but involve mutual inductance and are intended to be used with doubly wound coils. For accuracy greater than 1% such coils are very undesirable on account of the electrostatic capacity, and should not be used. Methods 9 and 10 are for comparing mutual inductance with capacity. Method 12 was investigated by Mr. Hemmings.

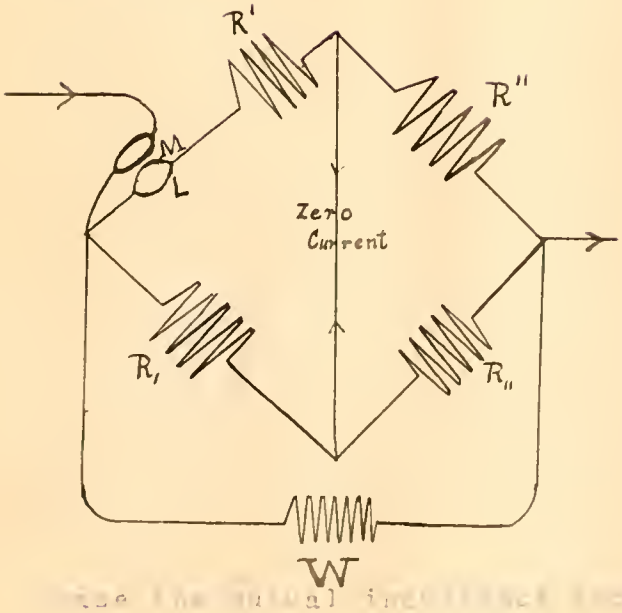
VI.

Methods 13.... 24.

The rest of the methods require at least two adjustments; first, the Wheatstone bridge must be balanced with direct currents, then adjustment made with alternating current so that the current through the bridge or is zero. Of these methods, 13 and 14, are for comparing two inductances, while methods 15 and 16, are for comparing inductance and capacity. However, method 15 requires first balancing the bridge with direct currents [illegible]

simultaneous adjustments with alternating currents; and Method 2c requires many resistances of known ratio. Method 1f is for the measurement of $\frac{L}{M}$ but is professor Nichols points out;"It is difficult to apply as two resistances must be adjusted, and the adjustment will only hold while the current period remains constant". These methods did not seem worthy of trial.

Method 1g is for the determination of $\frac{L}{M}$ where L and M belong to the same coil. This method is simpler than most of the others and seemed rather more promising, so it was given a trial. The diagram of the method is shown in the figure .



The bridge is first balanced with direct currents giving

$$R' R_{,,} = R_{,} R''.$$

Then with alternating current W is adjusted until there is zero current through the bridge arm as shown by an electrodynamometer or telephone. Then if $\frac{L}{M}$ is the ratio the mutual inductance should not; counterbalance but reinforce, so that the currents from electromotive forces in those in each arm of the bridge. Varying the current

through R' by I' etc,

$$R_1 I_1 = R' I' + i p L I' - i p M (I' + I_1 + I_w)$$

also $$R_1 I_1 = R' I'$$

and $$\frac{I'}{I_w} = \frac{M}{R' + R''}$$

$$\therefore \quad \frac{p M (I' + I_1 + I_w)}{I'} = p L$$

hence $$\frac{L}{M} = 1 + \frac{R'}{R_1} + \frac{R' + R''}{M}$$

The method was set up so that by means of a switch the resistance W could be replaced by the battery, and the electrodynamometer by the galvanometer. The method proved very unsatisfactory. Using the electrodynamometer in the bridge arm, its coils being connected in series, W could not be adjusted closer than 3Ω. Different values of the resistances were tried but the result was about the same. The electrodynamometer was then replaced by a telephone. It was then possible to adjust to a point to cut out the noise of the dynamo interfered somewhat with the use of the telephone. The conclusion was that the method is not sufficiently sensitive to be of value.

Method 10 is also a method for the comparison of $\frac{1}{2}$ inductances i.e. one of a double wound coil.

Method 10 was also rejected as unsuitable for double wound inductances.

to merit the construction of such a coil.

Method 22 is Carey Foster's method adapted to alternating currents. This method was rather thoroughly investigated by Heydweiller*.

Method 23 requires two simultaneous adjustments with alternating currents, and Method 24 is only of special use in comparing two doubly wound coils.

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* Wied. Ann. 43, 1894, p. 499.

SUMMARY.

With the exception of methods 14, 3 and 25 the methods tested are of little value, their chief defect being a lack of sensibility. Method 14, or more properly, the Osterbyck method, is without doubt the best of the methods for the absolute measurement of self-inductance in terms of resistance. It is easy to apply and if proper care be exercised, results should not differ from the mean by more than 2/10%. Greater accuracy than this is undoubtedly attainable, and ought not to be a matter of any great difficulty, for in the work here described the frequency determination could have been improved, and the resistances much more accurately measured. Method 25 is better than the Rayleigh method as ordinarily used, is very easy to work and has a very simple formula, involving little calculation. Its accuracy is probably limited to about 1%. Method 3 for comparing self-inductance with capacity is very sensitive and was found by Mr. Potts to be very satisfactory.

APPENDIX.

Professor Rowland, shortly before his death, outlined a method for the construction of tables whereby knowing the material and diameter of the wire, and thickness of insulation, a coil of given self-inductance could be wound, its dimensions being calculated from the tables, or knowing the dimensions and size wire it would be possible to foretell the self-inductance. It was decided to carry out this work, construct the tables, wind some coils according to them and so determine their value.

Maxwell* gives the following formula for the self-inductance of a coil

$$L = 4\pi n^2 a \left(\log_\epsilon \frac{8a}{b} - 2 \right)$$

where n is the number of turns, a is the mean radius of the coil, and b is the geometrical mean distance of the coil from its axis. If the cross section of the coil is a square of side s, then according to Maxwell, b = 0.44 S. Substituting this value in the formula

$$L = 4\pi n^2 a \left(\log_\epsilon \frac{8a}{.44s} - 2 \right)$$

$$= 4\pi n^2 a \log_\epsilon \left(2.422 \frac{a}{s} \right)$$

* Electricity and Magnetism. §716

Let n be the number of turns per sq. cm., therefore $n = \cdot\, s^2$

Hence, $L = \frac{4\pi}{10^9} N^2 \cdot A\, n \log_e \left(2.422 \frac{a}{s} \right)$ henrys.

If now we make $a = 2s$, which for a coil of square cross-section gives for a given length of wire the maximum self inductance,

$$L = \frac{0.9939\, N^4\, \pi\, a^5}{10^9 \cdot 2^{7/3}}$$

Hence

$$a = \frac{L^{1/5} N^{-2/5} \left(\frac{2^{4/3}}{\pi} \right)^{1/5} \cdot 10^{9/5}}{0.9939} = \frac{0.9568\, L^{1/5} N^{-2/5} (10)^{9/5}}{0.9939}$$

$$= 60.738\, L^{1/5} N^{-2/5}$$

Also,

$$n = N s^2 = \frac{Na^2}{4} = \frac{(60.738)^2 L^{2/5} N^{1/5}}{4}$$

$$= 922.28\, L^{2/5} N^{1/5}$$

Similarly, the total resistance, R, has the following value, in which r is the specific resistance and d the diameter of the wire;

$$R = \frac{N \cdot 2\pi \cdot a\, r}{\pi \left(\frac{d^2}{4} \right) 10^9} = \frac{N \cdot s^2 2\pi \cdot a\, r}{\pi \left(\frac{d^2}{4} \right) 10^9} \text{ ohms}$$

Let $\rho = \frac{4}{\pi} \frac{r}{10^9} \left(\frac{N}{d^2} \right)$; $V = 2\pi \cdot a\, s^2$

Hence $R = \rho V$

f ...ow a = 2 s, v = $\frac{1}{2}a^3$;

...d, if we substitute for e its value v

$$V = \frac{\pi}{2}\left[\quad 0.738 \quad L^{3/5} \cdot \quad ^{-6/5} \right]$$

Specifications for test coils X Y & Z.

$L \propto N^2 V^{5/3}$ $\quad a \propto L^{1/5} N^{-2/5}$ $\quad V \propto L^{3/5} N^{-6/5}$

B.&S. Gauge No.	Double Cotton Insulated Diameter cms	(Diameter)^2	N	N^2	$N^{-\frac{2}{5}}$	$N^{-\frac{6}{5}}$	$N^{\frac{1}{5}}$	$N^{\frac{3}{5}}$	S	$a = \frac{6.014 L^{\frac{1}{5}} N^{-\frac{2}{5}}}{a}$	$V = \frac{2\pi a s^2}{V}$	n total turns	Length of wire, feet	Resistance of Coil R ohms.	Weight insulated lbs.	Internal Diameter $2a - S$	External Diameter $2a + S$	Self-Inductance L
16	.1499	.02246	44.5	1982.7	21906				5.79	11.583 $L = \frac{1}{4}$ turn		1493	4277			17.37 cms	28.91	$\frac{1}{4}$ henry X
18	.1219	.01486	67.3	4529.3	.18569	.006403	2.3206	12.497	5.64	11.278 $L = 1$ henry	2253.3	2140	4974	31.8	9.3	16.92	28.20	1 "
22	.0851	.007240	138.1	19077.1	.13928	.002702	2.6795	19.238	4.23	8.458 $L = 1$ henry	950.91	2471.4	4809.	69.6	9.3	12.69	21.15	1 Y
27	.0556	.003457	289.3	83677.2	.10363	.001113	3.1064	29.977	3.15	6.293	391.34	2863.	3714.	191.3	2.6	9.44	15.73	1 Z

Henry Dickinson Hill was born in Baltimore, September 9, 1879. He was educated in various private schools of Baltimore. He matriculated in the Johns Hopkins University in 1897, and was admitted to the degree of A. B. in June 1900. He was since pursued advanced work in Physics, Mathematics and Chemistry.